Reduktionsformel für:

$$I = \int_0^{\pi/2} sin^n(x)\,dx =$$

$$-\left[cos(x)sin^{n-1}(x)\right]_0^{\pi/2} + (n-1)\int_0^{\pi/2} cos^2(x)sin^{n-2}(x)\,dx =$$

$$(n-1)\int_0^{\pi/2} (1-sin^2(x))sin^{n-2}(x)\,dx =$$

$$(n-1)\int_0^{\pi/2} (1-sin^2(x))sin^{n-2}(x)\,dx =$$

$$(n-1)\int_0^{\pi/2} sin^{n-2}(x) - sin^n(x)\,dx$$

$$\Rightarrow I = (n-1)\int_0^{\pi/2} sin^{n-2}(x)\,dx - (n-1)\int_0^{\pi/2} sin^n(x)\,dx$$

$$\Rightarrow \frac{I}{n-1} = \int_0^{\pi/2} sin^{n-2}(x)\,dx - \int_0^{\pi/2} sin^n(x)\,dx$$

$$\Rightarrow \frac{I}{n-1} + I = \int_0^{\pi/2} sin^{n-2}(x)\,dx$$

$$\Rightarrow \frac{n}{n-1} I = \int_0^{\pi/2} sin^{n-2}(x)\,dx$$

$$\Rightarrow I = \frac{n-1}{n}\int_0^{\pi/2} sin^{n-2}(x)\,dx$$

Damit gilt insbesondere:

$$\int_0^{\pi/2} sin^2(x)\,dx = \frac{\pi}{4}$$

Sowie:

$$\int_0^{\pi} sin^2(x)\,dx = \frac{\pi}{2}$$